
REPORT No. 202

THE SPARKING VOLTAGE OF SPARK PLUGS

By F. B. SILSBEE
Bureau of Standards

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I. INTRODUCTION

The study of this subject was originally undertaken at the request of the Army Air Service. In view of the intimate connection between this and other phases of the ignition problem studied under the auspices of the National Advisory Committee for Aeronautics, it is published as a technical report of the latter body to put on record the present state of knowledge of the voltage required to produce a spark across the gap of a spark plug in the cylinder of an internal combustion engine. This voltage is affected by a considerable number of conditions, some of which are little understood at present. In the following discussion the approximate range of each of these conditions will be indicated and at least the order of magnitude of its effect on the sparking voltage.

The importance of having an approximate knowledge of this voltage lies in the fact that it constitutes a sort of common meeting point in the performance of the engine, the spark plug, and the magneto (or coil). The voltage attained by the spark coil, and hence the stress to which all parts of the electrical system are subjected, is fixed solely by the sparking voltage. Thus any change in, for example, the compression ratio of the engine affects the operation of the magneto, or the most desirable setting of the spark plug points only in so far as it changes the sparking voltage. Information on sparking voltages is of importance also in connection with the safety gaps of magnetos, and to an even greater extent with the various forms of standard test gaps used in testing the performance of ignition apparatus.

A further point is the possible direct relation between the initial voltage of a spark and its igniting power. Work at the British (69)¹ National Physical Laboratory has shown quite definitely that of two very weak sparks of equal energy content, the one which has the greater sparking voltage is capable of igniting a less inflammable mixture. Certain writers (11), (48) have concluded from this that a high sparking voltage is therefore always desirable in a spark plug. It would seem, however, that the increased difficulty of producing any spark at all with such a plug would more than offset the benefit from the increased igniting power when a spark is produced. The experiments referred to in (69) were made with well-carburetted gaseous mixtures, and more data are needed when conditions are otherwise.

The range of variables covered by the present report will be only sufficient to cover adequately the limits met with in automotive practice. These are, roughly, gap lengths from 0.1 mm. to 6 mm., gas pressures from one-half atmosphere to 10 atmospheres, temperatures up to 700° C., and voltages up to 15,000 volts. For pressures or gap lengths materially outside this range the character of the electric discharge may become very different from those described in this report.

In the present report, after a brief account of the present accepted theory of the spark discharge, there will be given a tabulation of the principal variables affecting the sparking voltage and a detailed discussion of each. The report will then treat of the application of these facts to standard test gaps, auxiliary series gaps, and safety gaps, and will close with a general summary of the contents of the earlier sections.

The literature relating to spark gaps in general is very voluminous. The bibliography appended to this report contains such references as seem to have even a remote bearing on ignition work, arranged chronologically under a number of principal topics. The numbers in parentheses scattered through the text of the report refer to references in the bibliography covering the particular phase under discussion.

¹ Reference is made by number (italic) to "Bibliography of Sparking Voltages."

II. THEORY OF THE SPARK DISCHARGE

A single spark from a magneto or induction coil can be analyzed into an extended series of events, some of which occur slowly enough to be studied by the oscillograph or by a rotating mirror, while others are so rapid as defy direct observation and can only be inferred from indirect observation or from theory. The total sequence of events is as follows (70): After the primary contact points of the usual type of ignition circuit separate, the voltage at the spark plug gap begins to rise very rapidly, the rate being of the order of 50,000,000 volts per second. This continues, however, for only about a ten-thousandth of a second until the sparking voltage of the gap (say 5,000 volts) has been reached. The gas in the gap then suddenly "breaks down" (that is, it becomes a relatively good conductor of electricity) and a very large current flows across the gap. The magnitude of this first rush of current is determined by the voltage at which breakdown occurs, by the electrostatic capacity of the secondary wiring, and by the very small inductance of the secondary leads. This first discharge probably alternates in direction several times during the next few millionths of a second but rapidly dies down to a very much lower value (say 0.05 ampere) which is fixed mainly by the magnetic circuit and the number of turns of the secondary winding. This current dies away more gradually, the rate depending in part upon the length and proportions of the spark gap and in part upon the initial supply of energy, until (after about 0.005 second) the current suddenly stops. The subject of the present report is the value of voltage at which the gap breaks down and this value depends mainly on the dimensions, density, etc., of the gas in the gap and only slightly if at all on the ignition system supplying the voltage.

The breakdown of the spark gap under electric stress (at a definite voltage) is somewhat analogous to the failure of a solid test piece under a definite value of mechanical stress. The viewpoint afforded by this analogy is satisfactory and sufficient for many problems connected with sparking voltages. However, the development of the electron theory of matter and in particular of gaseous ions has given us a much more intimate and detailed picture of the process by which the gas changes at a certain definite voltage from an almost perfect insulator into a fairly good conductor of electricity. This theory has been developed by a large number of physicists. Very good summaries are given in the books of Thomson (3) and Townsend (8), and the chapter on this subject by Stark (71) in Winkelmann's "Handbuch der Physik." The first definite applications of the electron theory to the spark discharge were given in earlier articles by these authors (40), (42).

The essential features of the electron theory of sparks are as follows: (a) There exists in the air a considerable number (several hundred per cubic centimeter) of electrically charged particles called ions. (b) These move under the electric attraction from the electrodes and constitute the carriers of the electric current by moving to the electrodes and giving up their charges. (c) When the electric force in any region exceeds a certain critical value (about 30,000 volts per cm. at normal density) these ions may be accelerated to such a speed that on colliding with an electrically neutral gas molecule they split it into a fresh pair of ions, one being charged positively and the other negatively. (d) If the number of new ions thus produced in a given time exceeds the number swept out of the gap and discharged at the electrodes, then the number present will increase indefinitely and in geometric progression so that an enormous current will soon flow and be accompanied by the rapid dissipation of energy as heat, light, and noise which we call a spark.

III. TABULATION OF THE VARIABLES

The variables which may affect the sparking voltage of an engine spark plug may be tabulated as follows:

1. Gap length and shape of electrodes.
2. Gas density at the instant of ignition, which depends upon gas pressure and temperature which in turn depend upon spark advance, throttling both in induction system and at throttle valve, compression ratio, altitude.

3. Electrode temperature which in turn depends upon electrode size and shape, and gas leakage through the plug, as well as upon all the engine conditions listed under 2.
4. Polarity.
5. Electrode material and surface.
6. Mixture ratio.
7. Turbulence.
8. Rate of application of voltage and initial ionization.

The effect of items 1 and 2 have been the subject of a very great amount of research so that relatively little remains to be done on them. Item 3 has been the subject of recent study (39) and is probably the principal cause of uncertainty in predicting sparking voltages because of our lack of knowledge of the temperature of spark plug electrodes in operation.

The effects of items 4 to 7, inclusive, are of relatively little importance in automotive work. Item 8 and related topics have been the subject of considerable study and numerous controversies which have not as yet been satisfactorily settled.

IV. GAP DIMENSIONS

The relation between the sparking voltage and the shape and spacing of the electrodes has been the subject of a very great amount of experimentation. The first results obtained in absolute units were those of Sir William Thomson (Lord Kelvin) in 1860 (14), and were followed by a series of papers (17), (18), (19), (20), (23) extending to the recent work by the large American electric corporations (24), (25), (26) leading to the standardization of the sphere gap as a means for measuring high voltage. A summary of the work as applied to high voltage measurement is given by Peek (10) and by Toepler (4) and a more general summary by J. J. Thomson (3).

The work as a whole shows that air in bulk (i. e., as tested between parallel plates a considerable distance apart) can withstand a certain definite electric stress (30,000 volts per cm. at normal density). For other forms of electrodes two modifying effects enter so that the sparking voltage is not strictly proportional to the gap length. The first effect is due to the nonuniform space distribution of the electric force, and the second, which occurs in short gaps, arises from a lack of space in which ionization can occur.

The first effect is exemplified in long gaps between small or pointed electrodes. In such a gap the local intensity of the electric field is much greater near the electrodes and the gas near them will break down when the average voltage gradient over the whole gap is much less than the critical value. A greater stress is then put on the central part and it too may then fail. In extreme cases it is possible for the locally overstrained air to break down without imposing enough additional stress on the remainder of the air to break it down in turn. This condition produces the brush or corona discharge instead of a complete spark. The proportions of spark plug electrodes are very seldom such as to produce this effect, though corona often occurs at other parts of the secondary wiring of ignition systems where sharp points are present.

The effect of a nonuniform electric field can be allowed for in the computation of sparking voltage if the shape of the electrodes is such that the distribution of electric force can be computed mathematically. This can be done for spheres (1, 5, 7) parallel wires (10) and concentric cylinders (10) but the more complex shapes used for spark plug electrodes can not be handled mathematically. The maximum electric stress (or voltage gradient) in any gap is given by

$$g = \frac{v}{s} f \text{ volts per centimeter (12)}$$

where

g = maximum gradient

v = voltage applied to gap

s = spacing in cm

f = a factor depending on the shape of the electrodes.

For parallel planes $f = 1$.

$$\text{For concentric cylinders } f = \frac{s}{r \log_e \frac{R}{r}}$$

where R = radius of outer cylinder
 r = radius of inner cylinder

$$\text{For parallel wires } f = \frac{s}{2r \log_e \frac{s+2r}{r}} \text{ approx.}$$

where r = radius of wire.

$$\text{For equal spheres } f = \frac{s}{R} + 1 + \sqrt{\left(\frac{s}{R} + 1\right)^2 + 8}$$

For most of the electrodes used in spark plugs f probably does not differ so very greatly from 1.

The second reason for the departure of sparking voltage from direct proportionality with the gap length is an apparent increase in the specific dielectric strength of air (voltage gradient required for breakdown) in very short gaps. This increase was first attributed to a condensed layer of air close to the electrodes but a more probable explanation is that with the short gaps in which this effect is important, the space in which a given ion can produce others by collision is so short that it does not collide with enough molecules to supply the increase in ionization necessary for a spark. This cramping effect is also produced by curvature of the electrode

surfaces and tends to some extent to counterbalance the effects of nonuniformity previously discussed. The net result of these two effects is that the shaping of the spark plug electrodes has relatively little effect on the sparking voltage. This is shown by Figure 1 which gives the sparking voltages observed at the Bureau of Standards on 25 different spark plugs chosen to cover as wide a range of shape as possible. The solid curve corresponds to data by Paschen (28) on a gap between spheres 0.50 cm. in diameter. It will be noted that practically all the shapes of plug electrodes give sparking voltages within ± 20 per cent of the curve. Similar data on the sparking voltages of a variety of plugs at atmospheric pressure and also at 96 lb./in.² pressure have been reported from the British National Physical Laboratory (46).

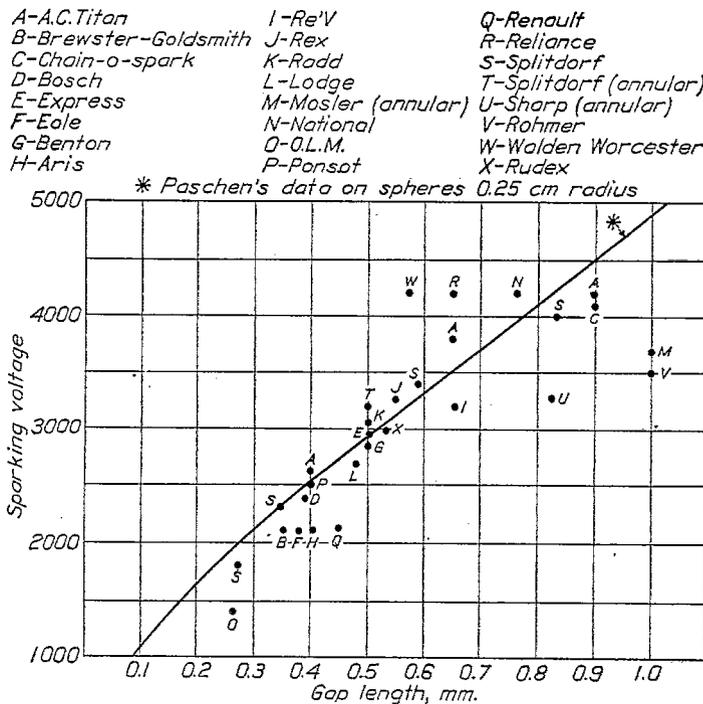


Fig. 1.—Sparking voltage in air at normal density (20° C., 760 mm.) of the various spark plugs listed in Table I

V. GAS DENSITY

The general fact that increased pressure hinders and rise of temperature assists the discharge of electricity has long been known. As far back as 1834 Harris (36) by heating at constant volume the gas around a spark gap showed that the breakdown voltage was not affected by such heating, which increased both the temperature and pressure but left the gas density unchanged. The results of a number of observers (16, 28, 31, 33, 34) have shown that the voltage increases linearly with an increase in pressure up to 10 atmospheres, but increases less rapidly at higher pressures. The effect of temperature has been less often investigated, but work done some years ago (35) has shown quite definitely that (up to five times normal density) it is the density of the gas which is of importance and that pressure and temperature changes in the gas affect its breakdown voltage only to the extent that they affect the density. In 1889

Paschen (28) stated a general relation (which, however, is only approximately true)—that the sparking voltage depends only on the product of the spark length and density. As a result of very careful measurements at densities from one-half to one atmosphere, Peek (10) has developed a rather elaborate formula for sphere gaps.

The simple linear relation, however, $E_s = E_1(1 + K(\delta - 1))$ expresses the observed data fairly well and is more convenient to use. Here

δ = density of air relative to normal temperature and pressure.

E_1 = sparking voltage at relative density = 1.

E_s = sparking voltage at relative density = δ .

K is a constant which varies from 0.5 to 0.7 for different shapes of electrodes, the larger values corresponding to blunter electrodes.

The average gas density at the instant of firing in an engine cylinder depends upon the throttle opening, spark advance, compression ratio, and for ordinary aviation engines, altitude, hence each of these variables affects the sparking voltage. Thus, for example, Figure 2 shows the effect of spark advance upon the voltage required to produce a spark. In the experiments corresponding to Figure 2 the engine (a 1-cylinder Liberty) was kept running by one spark plug, while another plug was connected to a source of high voltage which could be easily controlled and measured and which was applied to the plug for a very short interval at any desired point in the engine cycle. It will be noted that over the range of possible spark settings from 20 to 40° advance on the crank shaft (10 to 20° on the cam shaft), the voltage varies by 25 per cent. After the mixture has been ignited but before combustion is complete there are very great variations in density between various parts of the burning mass of gas, and consequently the results of successive measurements at times later than "dead center" are very discordant.

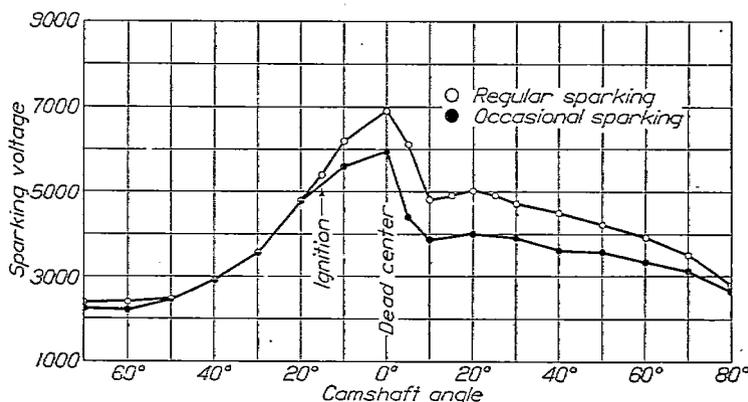


Fig. 2.—Voltage required to produce a spark at various times during an engine cycle. Data obtained in a 1-cylinder Liberty engine operating at part throttle

A change of throttle setting also changes the gas density at the instant of ignition, but it also changes the mean operating temperature of the electrodes very decidedly, and consequently introduces changes in sparking voltage as a result of changes of temperature as well as of density. This effect will next be considered.

VI. ELECTRODE TEMPERATURE

While an internal-combustion engine is operating, the spark plug electrodes are at considerably higher temperature than the fresh charge of gas, even after the latter has been mixed with the exhaust gases in the clearance volume and compressed adiabatically. Under conditions of preignition, temperature readings as high as 900° C. have been obtained on thermocouples imbedded in the central electrode of a spark plug,² and the "blueing" effect often noticed on the surfaces of the electrodes indicates the occasional existence of very high temperatures. Probably temperatures of 500 to 600° C. are more nearly normal for aircraft engines.

A few experiments (37, 38) have been made in the past on the effect of electrode temperature on sparking voltage and in 1902 Stark (41) suggested the existence of such an effect on theoretical grounds.

Recent experiments (39) have shown that this effect is of considerable importance. The measurements were made on a spark gap between a cold brass sphere 1 cm. in diameter and the tip of a thermocouple which could be heated by a small electric furnace. A blast of air play-

² The plug in question was not used for ignition during this measurement.

ing on the gap insured the maintenance of normal atmospheric density in the gas in the gap. When the negative electrode was heated the sparking voltage was materially reduced, the change amounting to 50 per cent at 700° C. When the positive electrode was heated the change was somewhat less (20 to 30 per cent) but still very definite.

As suggested by Stark (41) it is probable that this effect is the result of the existence of a thin layer of heated gas next to the hot electrode which is not swept away by the air blast nor by the turbulence in the engine. The effect of reduced density previously discussed would cause this thin layer to be overstressed, and it would then afford an effective ionizing region and cause the entire gap to break down.

Since the temperature of the spark plug is affected by a great number of factors, many of which are unknown, it is evident that this temperature effect prevents the computation of the sparking voltage in any particular case. Obviously, however, any change which increases the temperature of the spark plug such as greater throttle opening, higher compression ratio, lower altitude, greater gas leakage through the plug or changes of electrode shape will tend to correspondingly decrease the sparking voltage, and vice versa. There seems to be some slight possibility that this effect may be useful as a means of estimating plug temperatures, or at least of indicating the liability of a plug to heat excessively and cause preignition. Table I gives in the last column the ratio of the voltage required to produce a spark in an engine to that required in air at normal density and room temperature. The engine measurements were made in a 1-cylinder Liberty, at part throttle and 25° spark advance with a rather rich mixture. An approximate estimate of the increase in density due to compression would indicate an increase in voltage by a factor of 2.47. The plugs listed in Table I fall into two groups, one having on the average the ratio 1.57, while the other has 2.11. The plugs in the former group are of such construction that high temperature would be expected, while those in the latter group would be expected to run decidedly colder.

TABLE I

Make of plug	Type of gap	Gap length mm.	Volts in engine	Volts in engine
				Volts in air
Herz.....	Ponsot disk.....	0.40	3,500	1.40
Express.....	Wire end on.....	.50	4,200	1.40
O. L. M.....	Annular.....	.26	2,200	1.57
National.....	Parallel wires.....	.76	6,600	1.57
Bosch.....	3-point.....	.40	8,900	1.62
Champion.....	Wire end on.....	.56	5,600	1.65
Aris.....	Wire end with corrugations.....	.35	3,700	1.70
Mean.....				1.57
Re V.....	Wire to shell.....	.65	5,900	1.85
Bethlehem de luxe.....	Wire end on.....	.38	5,000	1.92
Bethlehem.....	Ponsot disk.....	.48	4,900	1.96
Sharp.....	Annular.....	.76	6,600	2.00
Splitdorf.....	do.....	.54	6,500	2.03
Benton.....	do.....	.60	5,900	2.04
Lodge.....	do.....	.49	5,600	2.08
Mosler M-5.....	Wire end on.....	.30	4,800	2.18
Eole.....	Lozenge to shell.....	.36	4,800	2.18
Brewster-Goldsmith 1-A.....	Annular.....	.40	4,600	2.19
Rudex.....	Wire to shell.....	.53	6,700	2.23
Brewster-Goldsmith 3-X.....	Wire end on.....	.51	4,600	2.24
Renault.....	Points to shell.....	.48	4,800	2.26
A. C. Titan.....	Crossed wires.....	.51	8,000	2.36
Mean.....				2.11

Curve II, Figure 3, shows the observed sparking voltages at upper dead center of a plug of a conventional type in a 1-cylinder Liberty engine operating at 1,200 R. P. M. with various throttle settings. The change of voltage with B. M. E. P. is slight because the changes in gas density and electrode temperature affect the sparking voltage in opposite directions. The gas density as estimated from the compression pressures given by a check valve indicator is shown in Curve V, and the mean temperature throughout the cycle of the electrode of a somewhat similar plug is shown by Curve III. (This latter curve was obtained from the indications of a thermocouple inserted in the central electrode of a Bosch plug which was not sparking.)

Curve I.—Voltage at various mixture ratios with constant throttle setting. Values with rich and lean mixtures are indicated by R and L, respectively, and practically coincide.

Curve II.—Voltage at various throttle openings with constant carbureter setting.

Curve III.—Temperature of plug electrode at various throttle settings with constant carbureter setting.

Curve IV.—Temperatures at various mixture ratios richer than the maximum power mixture, with constant throttle setting corresponding to point A. Point L indicates temperature with lean mixture and same throttle setting as A.

Curve V.—Density of gas at upper dead center at various throttle openings as estimated from check valve readings.

Curve I shows the slight increase in sparking voltage which occurs when, with constant throttle setting, the mixture is made either richer or leaner than that corresponding to maximum power. In this case the gas density is practically unchanged, but the electrodes are considerably cooled, as is indicated by Curve IV, which gives the temperature of the Bosch plug electrode at various mixture ratios. The data were obtained at the constant throttle setting corresponding to point A on Curve III. Point L indicates the temperature when the B. M. E. P. was reduced by leaning the mixture. The fact that the sparking voltage (Curve I) is the same with both very rich and very lean mixtures, in spite of the fact that the plug is somewhat cooler in the former case, is probably to be accounted for by the lower breakdown voltage of the gasoline vapor referred to below.

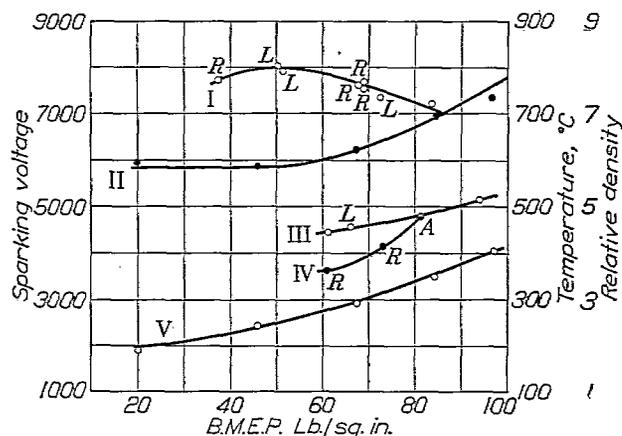


FIG. 3.—Sparking voltages and temperatures in a 1-cylinder Liberty engine at 1,200 F. Curves I and II are values for sparking voltage. Curves III and IV are values for temperature at upper dead center. Abscissae for all curves are values for B.M.E.P. (Lb./sq. in.)

VII. POLARITY

In the case of a spark gap with the two electrodes alike in all respects, the sparking voltage will be the same regardless of which terminal is charged positively and which negatively. If the two electrodes are different, however, either in shape or in temperature, there will in general be a difference in the sparking voltage for the two polarities. This is, however, usually slight unless the difference between the electrodes is very marked.

The average difference in the sparking voltage on the two polarities as measured at normal pressure and temperature on an assortment of spark plugs of very varied shape was 14 per cent. In an experimental gap between a cool sphere and a rather pointed electrode heated at 500°C. the voltage with the hot point as anode was as much as 30 per cent higher than with the reverse polarity.

It is probable that in most spark plugs the central electrode operates at an appreciably higher temperature than the side electrode and hence in installing a battery ignition system it is desirable to make the central electrode the negative terminal of the secondary circuit. In the case of a magneto, however, alternate sparks are of opposite polarity so that there is no preference in making connections.

VIII. ELECTRODE MATERIAL AND SURFACE

A number of experimenters have found that the sparking voltage does not depend upon the metal of which the electrodes are made, with the possible exception of aluminum and magnesium (3, 48, 65, 67). It is possible that indirect changes may be produced because of a difference in the temperature attained in operation but such effects are probably very small. (See also 68.)

Experiments³ have indicated that when the electrode surface is wet with oil or gasoline the voltage is increased by from 10 to 20 per cent. If a drop of oil bridges the narrowest part of the gap between the electrodes the spark takes place through the air around the surface of the

³ Bureau of Standards, 1921.

drop. This requires an increase in voltage which may vary from 80 per cent in a long gap to 300 per cent in a short gap, and will of course, depend on the shape and arrangement of the electrodes. While the plugs referred to elsewhere were being run in a Liberty engine very little effect from oil could be detected even at starting with the engine cold. At rare intervals, however, the volt-meter indicated excessive voltages amounting to a 100 per cent increase. This was probably the result of oil but may have been an extreme case of the "over-shooting" effect described below.

IX. MIXTURE RATIO

While considerable work has been done on the relative electric strength of various fixed gases such as air, hydrogen, CO₂, etc. (28, 31, 34), the only data available on spark voltages in gasoline vapor mixtures are those obtained at the British National Physical Laboratory (69). These indicate that over the range of explosive mixtures the breakdown voltage may be represented by the empirical formula

$$V = V_0(1 - 0.6\rho)$$

where V_0 = sparking voltage in pure air.

ρ = weight of gasoline divided by weight of air filling space at same temperature and total pressure as the mixture.

The linear variation of voltage with percentage of vapor is corroborated by the experiments of Hayashi (34) who found that the sparking voltage in mixtures of hydrogen and nitrogen could be accurately computed as a simple additive property of the composition of the mixture.

Since the normal value of ρ is only about 0.07 and some of this gasoline may be in liquid rather than vapor form the actual change in voltage as compared with that of the gap in air is only a few per cent. Rough attempts were made at the Bureau of Standards to detect this effect in an actual engine by observing the sparking voltage for various settings of the carburetor needle valve. Although other conditions were kept as nearly constant as possible, the changes in mixture caused such large changes in the operating temperature of the plug as to nearly mask any direct effect arising from the presence of the gasoline vapor. Consequently the actual sparking voltage as shown by Curve I, Figure 3, was slightly increased by varying the mixture ratio to either side of that giving maximum power.

X. TURBULENCE

From purely theoretical reasons it might be expected that a strong current of gas relatively free from ions blowing across a spark gap would interfere with the accumulation of ions necessary to produce a spark. In the case of a gap between a sharp point and a plane a blast of air along the point very definitely increases the sparking voltage for one polarity and thus produces a rectifying effect. Young and Warren (48, p. 357) have observed changes of 50 per cent in sparking voltage resulting from an air blast. On the other hand, in connection with the study of electrode temperature effects, careful measurements were made of the sparking voltage in an air stream at velocities up to about 4,000 cm. per second but no change was observed except what could be attributed to change of temperature resulting from a cooling of the electrodes. An explanation of this discrepancy may lie in a difference in the rapidity with which the sparks followed one another. If this was high in the British experiments (value is not given) the voltage at slow airspeeds may possibly have been reduced by residual ionization left in the gap by preceding sparks, while at faster airspeeds the spark occurred in fresh air.

XI. RATE OF APPLICATION OF VOLTAGE

Perhaps the greatest source of confusion, inaccuracy, and controversy in connection with observations of sparking voltage is the so-called "time lag" or "retardation" of the spark (52, 53, 54, 59, 60, 62, 63). This manifests itself in various ways depending upon experimental conditions usually either as a time interval or as an apparent increase of sparking voltage. The time intervals involved are negligible as such in automotive work but the increase in required voltage may well cause failure to spark at all.

If a continuous voltage slightly greater than the sparking voltage be applied to a spark gap a spark in extreme cases may not pass for several seconds or even minutes thereafter. In the case of an alternating voltage of commercial frequency where the voltage is near its maxi-

imum value for as much as 0.004 second at a time, the lag effects are not so very serious, particularly since the voltage is again applied on the next half wave after an additional interval of only about 0.004 second. The voltage from an ignition system, however, is near its peak value for only about 0.0001 second and the dead interval between peaks is usually 500 times this period. Consequently a relatively slight time lag will cause the plug to completely fail to spark. When an auxiliary spark gap is used in series with a spark plug the application of the voltage to the plug itself is still more rapid and may occur in less than 0.000001 second. Peek (10, 61) has studied the behavior of spark gaps with such suddenly applied voltages and finds that different types of gap vary greatly in the amount of "lag." Sphere gaps spark at substantially the same voltage as with more gradual applications but gaps between sharply pointed electrodes do not spark until the maximum suddenly applied voltage is very much (2 to 5 times) greater than the slowly applied sparking voltage. Work at frequencies up to 100,000,000 cycles per second (56) has shown lag effects even with sphere gaps.

The principal effects of this time lag when an ignition system is used as a source of voltage are (a) irregular firing unless the maximum voltage which the system will give greatly exceeds that which, at moments of least lag, will produce a spark, (b) an abnormally high sparking voltage which may be measured with the type of crest voltmeter, which indicates the maximum value of voltage applied to the gap. The amount of this increase in voltage depends on the ignition system as well as on the gap (64). It must be noted, however, that this lag is absolutely negligible as far as concerns any change in the point of the engine cycle at which ignition occurs.

The maximum secondary voltage from an ignition coil operating without any spark gap is proportional to the primary current. If such a coil is connected to a spark gap, and the current is gradually increased, a value will be reached, at which sparks will occur occasionally but not regularly. A crest voltmeter connected across the spark gap will indicate a voltage corresponding to the primary current. If the current is again increased somewhat the sparking will become more and more regular, and the reading of the crest voltmeter at first will continue to increase in proportion to the primary current. After the current has been raised sufficiently to give regular sparking, a further increase in primary current will still increase the voltmeter reading but to a less extent than corresponds to its proportionality with the primary current. Since the crest voltmeter indicates the greatest voltage applied to it during the five minutes preceding an observation, it must be concluded that the sparks lag at least occasionally until the voltage has risen considerably above the least value, at which a spark sometimes passes, before the gap breaks down, but that the gap always breaks down before the voltage has risen to the peak value which the given primary current is capable of producing. Since this last described condition of regular firing with an excess value of primary current is the normal condition of operation of an ignition system, it is evident that there is considerable of this "overshooting" effect present at all times, and its amount depends on the excess of primary current over that barely sufficient to fire the gap. The value of much experimental work on sparking voltages is limited by uncertainties due to this effect.

Another case in which this time lag manifests itself is where two spark gaps are connected in parallel. Such a situation arises, for example, when a previously calibrated spark gap is used to measure the sparking voltage of another gap, or in the safety gap of a magneto or the possible spark path over the outer surface of a spark plug insulator, both of which are in parallel with the normal spark plug gap. If two such parallel gaps differ markedly in time lag and a rapidly rising voltage impulse is applied to them, the spark will pass at the "quicker" gap, even if it is the longer of the two, and has, when measured alone, a decidedly higher sparking voltage than its "slower" companion. This emphasizes the caution which must be observed in making or interpreting experiments using such parallel gaps.

It was found very early (50) that these lag effects could be very greatly reduced by illuminating the spark gap and particularly the cathode with ultra-violet light. A similar reduction results from X rays, radium radiation, or any source which tends to increase ionization in the spark gap. A rather violent controversy (52, 53) arose over the question of whether the source of ionization actually reduces the sparking voltage or whether it merely reduces the lag effects. The preponderance of evidence seems (57, 58) to show that when the external source of ioniza-

tion is very powerful an actual reduction in voltage is produced, but the reduction in lag is nearly complete with much weaker sources of ionization.

The cause of this time lag has been the subject of considerable speculation (63, 64) but without very definite results. There is, of course, a certain interval required for the succession of collisions of ions with molecules to build up a sufficient density of ions in the gap to produce a spark. This time interval would be expected to be of the order of 0.00001 second, and it may well be that the lag effects noted by Peek (10) and Algermissen (56) are due to this. The atmosphere usually contains about 1,000 ions per cubic centimeter at all times, as a result of the radioactivity of the earth. In a gap as small as that of a spark plug, however, the volume of air which is under appreciable electric stress is only a few cubic millimeters, and it may well happen that no ion is present to start a spark during the short interval while an ignition voltage is actually applied to the gap. This absence of "casual ions" which could "trigger off" the spark is a second possible explanation of lag. It agrees with the observed fact that the lag is greater with point gaps than with electrodes of large radius where a greater volume of gas is subjected to the electric forces. Campbell (63) suggests still another rather mysterious source of lag (or as he terms it "hardness") which seems to depend on the surface condition of the cathode.

In the operation of a plug in an engine the gas is diluted with exhaust gases from the preceding cycle and these gases are probably highly ionized as a result of the chemical reaction just completed in them. Also the electrodes of the plug are hot and a certain amount of thermionic emission probably comes from them. As a result we should expect relatively little lag under these conditions. This is borne out by experiments at the Bureau of Standards, in which the crest voltage applied to the plug in actual operation was measured while the ignition system was supplied with various amounts of primary current. The average "overshoot" in voltage when the current was double that at which sparking just began amounted to only 7 per cent while in the open air with the plug cold the overshoot was 35 per cent, and when illuminated with an arc lamp it was 17 per cent. On the other hand, lag effects are often noted in the operation of magneto safety gaps, the flashover of spark plugs, and especially in standard test gaps. It is evident that there is much need for a more fundamental study of these lag phenomena in general.

XII. STANDARD TEST GAPS

In the testing of ignition apparatus both in the factory and in service, use is very frequently made of sets of standard spark gaps which are intended to duplicate the conditions of operation of the system either normally or more often with an arbitrarily chosen increase in severity. Such a set of test gaps is much cheaper and more convenient than any type of crest voltmeter and its further development is very desirable (49).

The requirements for such a type of gap are (a) ease of specification (i. e., few essential dimensions), (b) constancy of voltage (i. e., freedom from eating away of points), (c) freedom from time lag, (d) ease of setting (i. e., voltage should change only slowly with change of gap length). Items (a) and (b) are fulfilled very nicely by the sphere gap and it is the generally accepted standard (26) for high-voltage measurements. For the lower voltages used in ignition, however, the required gap length is very short and it is difficult to comply with item (d). Because of this recourse is had to gaps between pointed electrodes which satisfy (d) fairly well but at the expense of (a) and (b). A gap between two sharp points is very bad as regards item (c), however, and it is the universal practice to reduce the time lag troubles by the use of a third or "teaser" electrode which is insulated from the rest of the circuit and is separated from the high potential main electrode by a small gap (0.1 mm.). With such an arrangement, as the voltage on the main electrode rises, a tiny spark passes to the teaser electrode to charge it and its support to some intermediate potential corresponding to its intermediate position with respect to the two main electrodes. This small "teaser" spark produces ionization in the main spark gap and thereby prepares it for sparking at the slightly later instant at which the voltage has risen to the sparking voltage of the main gap. The usual explanation of the ionization of the main gap by the teaser spark is that some of the ions of the teaser spark move bodily into the main gap. Some recent experiments at the Bureau of Standards, however, indicate that the ultra-violet light from a very feeble teaser spark at a distance of several centimeters is sufficient to trigger off the main gap even when a direct transport of ions to the main gap is prevented by a barrier of transparent quartz.

TABLE II
SPECIFICATION FOR STANDARD 3-POINT GAP FOR IGNITION TESTING

- (a) Electrodes to be round wires of nickel alloy or platinum iridium.
 (b) Diameter of each wire to lie between 1.50 and 1.60 mm.
 (c) Each electrode to taper with an included angle of 45°:
 1. Tip of each electrode to be slightly blunted, the radius of curvature not to exceed 0.2 mm.
 2. Tip of teaser electrode to be sharp.
 (d) Axis of teaser electrode to be perpendicular to axis of main electrodes; tip of teaser to be 1 mm. back from tip of high-tension electrode; length of teaser gap to be between 0.1 and 0.2 mm.
 (e) Gaps complying with the above conditions have breakdown potentials approximately as follows:

Length of main gap	Breakdown potential
<i>mm.</i>	<i>Volts</i>
5.0	8,000
7.0	11,000

The voltage of the usual three-point gap depends considerably on the sharpness of the points and even with the teaser in operation there is considerable overshooting (as much as 25 per cent has been observed).

Other types of gap have been suggested for test purposes, such as the sphere gap and annular gap (43, 48), which seem to be somewhat less troubled by lag effects. They have not come into general use in this country as yet, and in the absence of any really satisfactory type of gap, the form described in Table II seems to be the most reliable.

XIII. SAFETY SPARK GAPS

Most magnetos are provided with a safety spark gap which is intended to discharge at some voltage well above the normal sparking voltage of the plug, but lower than that at which failure of the solid insulating materials used in the machine would be expected. Measurements on the safety gaps supplied with five different makes of magneto showed surprisingly uniform values of sparking voltage. The average was 12,000 volts and the separate machines differed less than 10 per cent from this value. Considerable "overshooting" was noted, however, and at times voltages 25 per cent greater than normal were required to produce a spark. Since the spark plug voltage under average conditions for aircraft engines is about 6,000 volts, it appears that safety gaps as now adjusted afford a margin of 100 per cent overvoltage.

In the case of battery and coil systems the total available energy at each instant of "break" is decidedly less than in a magneto, and also the design of the coil allows more space for solid insulation than is available in a magneto. Consequently safety gaps are not used and in case the high tension terminal becomes open circuited the coil is free to build up the maximum voltage of which it is capable with the given primary current. This value is usually from 10,000 to 15,000 volts, and hence is not materially different from that of the magneto safety gap.

XIV. SERIES GAPS

Relatively few experiments have been made on the sparking voltage of combinations of two or more spark gaps in series, although this condition arises in magnetos having "jump-spark" distributors and in cases where auxiliary series spark gaps (so-called "energizers," "intensifiers," or "transformers") are used. This condition also arises where a "2-spark" magneto fires two spark plugs in series.

The voltage required to spark two gaps in series is not in general the sum of the sparking voltages of the separate gaps but depends on the distribution of the total voltage between them. If one gap is shunted by a resistance, as is the case when a spark plug is even slightly fouled with carbon, practically the full voltage is concentrated on the clean gap and this latter will break down when the total voltage is equal to its sparking voltage. The full voltage is thus immediately applied to the fouled gap, which in turn will spark, provided its sparking voltage is also exceeded. This simple theory has been previously discussed (44, 45, 47), but there seems to be some evidence that the auxiliary gap is even more effective than this reasoning would account for (48). This may well be the result of electrical oscillations produced by the breakdown of the clean spark gap which in effect serve to apply nearly a double voltage to the fouled gap. This is a point which requires further investigation.

When both gaps are clean (i. e., free from parallel resistances) the division of voltage between the two gaps is determined by the electrostatic capacity in parallel with each. If these capacities happen to be inversely proportional to the sparking voltages of the corresponding

gaps then the total sparking voltage will, theoretically, be the sum of the sparking voltages of the individual gaps. Any other distribution of capacity will decrease the total voltage required to produce a spark. In a 2-spark magneto the capacity to the ground from one terminal is usually very much greater than that from the other.

XV. SUMMARY

The preceding discussion has shown that the sparking voltage of spark plugs depends upon a great number of conditions, some of which, such as electrode temperature, are so difficult to determine that the voltage in any given case can not be closely predicted. Various factors affect the voltage as follows:

1. Gap length increases the voltage approximately according to the curve of Figure 1.
2. Shape of electrodes may cause departures of ± 20 per cent from the curve of Figure 1.
3. Density of the gas increases the sparking voltage approximately linearly according to the equation

$$E_{\delta} = E_1 (1 + K(\delta - 1))$$

where δ = relative density

E_1 = voltage at $\delta = 1$

E_{δ} = voltage at $\delta = \delta$

K = constant = 0.6 approximately.

4. Heating the electrode above the temperature of the gas further reduces the voltage by amounts up to 50 per cent at 700° C. This reduction is greater when the heated electrode is negative.
5. Electrode material, mixture ratio, and turbulence have little direct effect on sparking voltage.
6. Erratic and large changes in voltage may be produced by time-lag effects, but these are less likely to occur under spark-plug conditions than in test gaps, safety gaps, etc.

A reasonable figure for the sparking voltage under average conditions in an aircraft engine at full power at sea level is 6,000 volts. Under extreme combinations of conditions it may rise to 10,000 or fall to 3,000 volts.

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